NOTE : DO NOT BREAK THE SEAL UNTIL YOU GO THROUGH THE FOLLOWING INSTRUCTIONS

COMMON ENTRANCE TEST - 2012

Question Booklet MATHEMATICS

Booklet No.

Roll No. | | | | | | |

(Enter your Roll Number in the above space)

Series **C** 401003

Time Allowed : 1.30 Hours

Max. Marks : 75

INSTRUCTIONS :

1. Use only BLACK or BLUE Ball Pen.
2. All questions are COMPULSORY.
3. Check the BOOKLET thoroughly.

   IN CASE OF ANY DEFECT – MISPRINTS, MISSING QUESTION/S OR DUPLICATION OF QUESTION/S, GET THE BOOKLET CHANGED WITH THE BOOKLET OF THE SAME SERIES. NO COMPLAINT SHALL BE ENTERTAINED AFTER THE ENTRANCE TEST.
4. Before you mark the answer, fill in the particulars in the ANSWER SHEET carefully and correctly. Incomplete and incorrect particulars may result in the non-evaluation of your answer sheet by the technology.
5. Write the SERIES and BOOKLET NO. given at the TOP RIGHT HAND SIDE of the question booklet in the space provided in the answer sheet by darkening the corresponding circles.
6. Do not use any **eraser, fluid pens, blades** etc., otherwise your answer sheet is likely to be rejected whenever detected.
7. After completing the test, candidates are advised to hand over the OMR ANSWER SHEET to the Invigilator and take the candidate's copy with yourself.

SEAL

(MATHEMATICS)

Choose the most suitable answer in the following questions :

1. In a triangle ABC, let $\angle A = \frac{\pi}{2}$ and $(a+b+c)(b+c-a) = \lambda bc$, then λ equals to

(1) 0 (2) 1 (3) 2 (4) -2

2. In a triangle, the lengths of the two larger sides are 10 and 9 respectively. If the angles are in A.P. then the length of the third side can be

(1) 4 (2) 5 (3) $6-\sqrt{6}$ (4) $5+\sqrt{6}$

3. If θ lies in the second quadrant and $3\tan\theta + 4 = 0$, then the value of $\sin\theta + \cos\theta$ is equal to

(1) $\frac{1}{5}$ (2) $\frac{2}{5}$ (3) $\frac{3}{5}$ (4) $\frac{4}{5}$

4. Let A and B be acute angles such that $\sin A = \sin^2 B$ and $2\cos^2 A = 3\cos^2 B$. Then A equals to

(1) $\frac{\pi}{4}$ (2) $\frac{\pi}{6}$ (3) $\frac{\pi}{3}$ (4) none of the above

5. If $\sin x \cos y = \frac{1}{4}$ and $3\tan x = 4\tan y$ then $\sin(x-y)$ equals to

(1) $\frac{1}{16}$ (2) $\frac{1}{8}$ (3) $\frac{3}{16}$ (4) $\frac{3}{4}$

6. If $A+B+C=\pi$ and $\sin C + \sin A \cos B = 0$ then $\tan A \cdot \cot B$ is equals to

(1) 0 (2) $-\frac{1}{2}$ (3) 1 (4) -1

7. If n is an even integer, then the value of ${}^nC_0 + {}^nC_2 + {}^nC_4 + \cdots + {}^nC_n$ equals to

(1) 2^n (2) 2^{n+1} (3) 2^{n-1} (4) 2^{2n}

8. The coefficient of the term independent of x in the expansion of $\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)^{10}$ is equal to

(1) 10 (2) 252 (3) 20 (4) 256

9. The range of the function $f(x) = \log_a x$, $a > 0$ is

(1) $(0, \infty)$
(2) $(-\infty, \infty)$
(3) $(-\infty, \infty) - \{0\}$
(4) none of these

10. The number of points at which the function $f(x) = \dfrac{1}{\log_e |x|}$ is discontinuous is

(1) 1
(2) 2
(3) 3
(4) ∞

11. The value of $\underset{x \to 0}{\text{Lt}} \dfrac{\sin(\pi \cos^2 x)}{x^2}$ is equal to

(1) $\dfrac{\pi}{2}$
(2) π
(3) $-\pi$
(4) 1

12. The function $f(x) = \dfrac{x}{1+x^2}$ decreases in the interval

(1) $(-\infty, -1] \cup [1, \infty)$
(2) $(-1, 1)$
(3) $(-\infty, \infty)$
(4) none of these

13. Let $f : \mathbb{R} \to \mathbb{R}$ be a function such that the third derivative of $f(x)$ vanishes for all x. If $f(0) = 1$, $f'(2) = 4$ and $f''(1) = 2$, then $f(x)$ equals

(1) $x^2 + 1$
(2) $x^2 + 2x + 1$
(3) $4x + 1$
(4) $x^2 - 2x + 1$

14. If $f'(x) = g(x)$ and $g'(x) = -f(x)$ for all x and $f(1) = 5$, $f'(1) = 4$, then the value of $f^2(1) + g^2(1)$ is equal to

(1) 25
(2) 16
(3) 41
(4) 9

15. The degree of the differential equation satisfied by the curves

$$\sqrt{1+x} - a\sqrt{1+y} = 1,$$

where 'a' is a parameter is

(1) 1
(2) 2
(3) 3
(4) none of the above

16. The probability that atleast one of the events A and B occurs is 0.5. If A and B occur simultaneously with probability 0.2, then $P(A^C)+P(B^C)$ is equal to

(1) 1.0 (2) 1.1 (3) 0.7 (4) 1.3

17. The following table gives the probability that a certain computer will malfunction 0, 1, 2, 3, 4, 5 or 6 times on any day:

No. of malfunctions x :	0	1	2	3	4	5	6
Probability $f(x)$:	0.17	0.29	0.27	0.16	0.07	0.03	0.01

Then mean of this probability distribution is

(1) 1.74 (2) 1.80 (3) 0.74 (4) none of these

18. For the married couple living in Jammu, the probability that a husband will vote in an election is 0.5 and the probability that a wife will vote is 0.4. The probability that the husband votes, given that his wife also votes is 0.7. Then the probability that husband and wife both will vote is

(1) 0.28 (2) 0.20 (3) 0.35 (4) 0.15

19. Let

$$f(x)=\frac{k}{1+x^2}, -\infty < x < \infty$$

be the probability density of a random variable. Then k equals to

(1) π (2) $-\pi$ (3) $\frac{1}{\pi}$ (4) 1

20. If mean and variable of a binomial variate X are 2 and 1 respectively, then the probability that X takes a value at least one is

(1) $\frac{1}{16}$ (2) $\frac{3}{16}$ (3) $\frac{5}{16}$ (4) $\frac{15}{16}$

21. Let A and B be two mutually exclusive events such that $P(A\cap B^C)=0.25$ and $P(A^C\cap B)=0.5$. Then $P\left((A\cup B)^C\right)$ is equal to

(1) 0.25 (2) 0.50 (3) 0.75 (4) 0.40

22. If the product of n positive real numbers is one, then their sum is

(1) $n+\frac{1}{n}$ (2) $n-\frac{1}{n}$

(3) $2n+\frac{1}{n}$ (4) never less than n

23. The sum of the series

$$5-\frac{10}{3}+\frac{20}{9}-\frac{40}{27}+\cdots$$

is equal to

(1) 1 (2) 2 (3) 3 (4) none of these

24. The sum of n terms of the series

$$\frac{3}{1^2\cdot 2^2}+\frac{5}{2^2\cdot 3^2}+\frac{7}{3^2\cdot 4^2}+\cdots$$

is equal to

(1) $\frac{n^2-2n}{(n+1)^2}$ (2) $\frac{n^2-2}{(n+1)^2}$ (3) $\frac{n^2+2n}{(n+1)^2}$ (4) $\frac{n^2+2}{(n+1)^2}$

25. If ${}^{m+n}P_2=90$ and ${}^{m-n}P_2=30$, then (m, n) is given by (m and n are positive integers)

(1) (8, 2) (2) (5, 6) (3) (3, 7) (4) (8, 3)

26. In the binomial expansion of $(a-b)^n$, $n\geq 5$, the sum of 5th and 6th term is zero. Then $\frac{a}{b}$ equals to

(1) $\frac{n-4}{2}$ (2) $\frac{n-4}{3}$ (3) $\frac{n-4}{5}$ (4) $\frac{n-4}{4}$

27. The value of $\tan\left(\cos^{-1}\frac{4}{5}+\tan^{-1}\frac{2}{3}\right)$ is equal to

(1) $\frac{17}{6}$ (2) $\frac{22}{15}$ (3) $\frac{16}{5}$ (4) $\frac{16}{9}$

28. If A is a skew symmetric matrix of order 3×3, then determinant of $|A|$ equals to

(1) 0 (2) 1 (3) 2 (4) -1

29. The trajectory of the differential equation

$$\frac{dx}{dt} = rx\left(1 - \frac{x}{k}\right), \; r > 0$$

is monotonically increasing if

(1) $x(0) < k$ (2) $x(0) > k$

(3) $\frac{k}{2} < x(0) < 2k$ (4) none of these

30. If $f(x) = (1+x)^n$, then the value of

$$f(0) + f'(0) + \frac{f''(0)}{2!} + \cdots + \frac{f^{(n)}(0)}{n!}$$

is equal to

(1) 2^{n-1} (2) $2n$ (3) n (4) 2^n

31. If $f : \mathbb{R} \to \mathbb{R}$ be a differentiable function such that $f(4) = 6$ and $f'(4) = 2$, then

$$\underset{x \to 4}{Lt} \frac{x f(4) - 4 f(x)}{x - 4}$$

is equal to

(1) 2 (2) -2 (3) 0 (4) 1

32. If $xe^{xy} = y + \sin^2 x$, then $\frac{dy}{dx}$ at $x = 0$ is equal to

(1) 0 (2) 1 (3) e (4) -1

33. Let $P(x, y)$ be a point on the curve $y^2 = 4x$ at which the tangent is perpendicular to the line $2x + y = -2$. Then the co-ordinates of the point P are

(1) (4, 4) (2) (4, −4) (3) (−4, 4) (4) (−4, −4)

34. Let a line makes an angle of 60° with each of the x and y axes. Then the angle made by the line with z-axis is

(1) 30° (2) 45° (3) 60° (4) none of these

35. The value of 'a' for which the volume of parallelopiped formed by the vectors $\hat{i}+a\hat{j}+\hat{k}$, $\hat{j}+a\hat{k}$ and $a\hat{i}+\hat{k}$ is minimum is

(1) $\frac{1}{\sqrt{3}}$ (2) 3 (3) −3 (4) 1

36. The angle between the two lines

$$\frac{2-x}{1}=\frac{y}{2}=\frac{z+3}{1} \text{ and } \frac{x-4}{4}=\frac{y-1}{1}=\frac{z-5}{2}$$

is

(1) 0° (2) 90° (3) 45° (4) none of these

37. If the foot of perpendicular from the origin to a plane is (1, 2, 3), then equation of the plane is

(1) $2x-y+z=3$ (2) $x+y+z=6$

(3) $x-y-z=-4$ (4) $x+2y+3z=14$

38. The length of perpendicular from the point (1, 0, 1) to the plane $3x+\sqrt{6}y+7z+6=0$ is

(1) 2 (2) 6 (3) 8 (4) 7

39. If the line $\frac{x-4}{1}=\frac{y-2}{1}=\frac{z-k}{2}$ lies in the plane $2x-4y+z=5$, then k is equal to

(1) 0 (2) 1 (3) 7 (4) 5

40. If A and B be two independent events such that $P(A)=\frac{2}{3}$ and $P(B)=\frac{1}{5}$, then $P(A\cup B)$ equals to

(1) $\frac{1}{15}$ (2) $\frac{11}{15}$ (3) $\frac{2}{15}$ (4) $\frac{13}{15}$

41. If $iz^3 - z^2 + z + i = 0$, then z lies on

(1) a circle with centre (0, 0) and radius 1
(2) a circle with centre (1, 0) and radius 1
(3) a circle with centre (0, 1) and radius 1
(4) a straight line

42. If $|z| = 1$ and $w = \frac{z+1}{z-1}$ (where $z \neq 1$), then Re(w) equals to

(1) $\frac{1}{|z-1|}$ (2) $\frac{1}{|z+1|}$ (3) $\left|\frac{z}{z-1}\right|$ (4) 0

43. If $f(x) = \sin\left(\log\left(\frac{\sqrt{16-x^2}}{2-x}\right)\right)$, then domain of $f(x)$ is equal to

(1) (–4, 2) (2) (–4, 4) (3) (–4, 4] (4) [–4, 4]

44. The real value of θ for which the expression $\frac{1+i\sin\theta}{1-2i\sin\theta}$ is a real number.

(1) $n\pi$, n is an integer (2) $2n\pi + \frac{\pi}{2}$, n is an integer

(3) $2n\pi - \frac{\pi}{2}$, n is an integer (4) $n\pi + \frac{\pi}{2}$, n is an integer

45. Let α and β be the roots of equation

$$x^2 - (a-2)x - a - 1 = 0$$

then $\alpha^2 + \beta^2$ assumes the least value if

(1) $a = 0$ (2) $a = 1$ (3) $a = -1$ (4) $a = 2$

46. For a real x, the equation

$$e^{\sin x} - e^{-\sin x} - 16 = 0$$

has

(1) one and only one solution (2) four solutions
(3) infinite number of solutions (4) no solution

47. The number of distinct real roots of

$$\begin{vmatrix} \sin x & \cos x & \cos x \\ \cos x & \sin x & \cos x \\ \cos x & \cos x & \sin x \end{vmatrix} = 0$$

in the interval $-\frac{\pi}{4} \le x \le \frac{\pi}{4}$ is

(1) 0 (2) 1 (3) 2 (4) 4

48. Let $a > 0$, $b > 0$ and

$$f(x) = \begin{vmatrix} x & a & a \\ b & x & a \\ b & b & x \end{vmatrix}$$

then which of the following statements is true?

(1) $f(x)$ has a local minimum at $x = \sqrt{ab}$

(2) $f(x)$ has a local maximum at $x = \sqrt{ab}$

(3) $f(x)$ has neither local minimum at $x = \sqrt{ab}$ nor local maximum at $x = \sqrt{ab}$

(4) none of the above

49. The system of homogeneous equations

$$tx + (t+1)y + (t-1)z = 0$$
$$(t+1)x + ty + (t+2)z = 0$$
$$(t-1)x + (t+2)y + tz = 0$$

has non-trivial solutions for

(1) exactly three real values of t (2) exactly two real values of t

(3) exactly one real value of t (4) infinite number of values of t

50. Matrix A is such that $A^2 = 2A - I$, where I is the identity matrix, then for $n \ge 2$, A^n is equal to

(1) $2^{n-1}A - (n-1)I$ (2) $2^{n-1}A - I$

(3) $nA - (n-1)I$ (4) $nA - I$

51. If A is a matrix of order n, then determinant $|-A|$ is equal to

(1) $|A|$ (2) $-|A|$ (3) $(-1)^n|A|$ (4) $n|A|$

52. The value of the integral

$$\int_{-\pi/2}^{\pi/2} \sqrt{1-\cos^2\theta}\, d\theta$$

is equal to

(1) 0 (2) 1 (3) 2 (4) –2

53. If

$$f(x) = \int_{1}^{x} \sqrt{4-t^2}\, dt,$$

then real roots of the equation $x - f'(x) = 0$ are

(1) ± 1 (2) $\pm\sqrt{2}$ (3) 0 and 1 (4) ± 2

54. If

$$f(x) = \log_e(1+x) - \log_e(1-x),$$

then the value of $\int_{-1/2}^{1/2} f(x)\, dx$ equals to

(1) 0 (2) 1 (3) $\frac{1}{2}$ (4) $-\frac{1}{2}$

55. Let $f:(0,\infty) \to \mathbb{R}$ and $F(x) = \int_{0}^{x} f(t)\, dt$. If $F(x^2) = x^2(1+x)$, then $f(1)$ equals to

(1) $\frac{5}{2}$ (2) 5 (3) $\frac{2}{5}$ (4) 2

56. The vectors $\vec{a} = \hat{i} + 4\hat{j} - 7\hat{k}$, $\vec{b} = \lambda\hat{i} - \hat{j} + 4\hat{k}$ and $\vec{c} = -9\hat{i} + 18\hat{k}$ are coplanar if λ equals to

(1) 0 (2) 1 (3) 2 (4) 3

57. Let $\vec{a}$ and $\vec{b}$ be two unit vectors such that $\vec{a} + 2\vec{b}$ and $5\vec{a} - 4\vec{b}$ are perpendicular to each other, then the angle between $\vec{a}$ and $\vec{b}$ is

(1) 30° (2) 45° (3) 60° (4) 90°

58. Let $\mathbb{R}$ be the set of real numbers and $f:\mathbb{R}\to\mathbb{R}$ be a function defined by $f(x)=x^2+4$. Then $f^{-1}(29)$ equals to

(1) ϕ (2) $\{5, -5\}$ (3) $\{4, -4\}$ (4) $\{2, -2\}$

59. The function $f:\mathbb{R}\to\mathbb{R}$ defined by

$$f(x)=\frac{e^{|x|}-e^{-x}}{e^{x}+e^{-x}}$$

is

(1) one-one and onto (2) one-one but not onto

(3) not one-one but onto (4) neither one-one nor onto

60. The sum of the coefficients of the polynomial $(1+x+x^2-4x^3)^{2149}$ is

(1) 1 (2) –1 (3) 2143 (4) 2156

61. If $z=1-i$, then principal value of $\arg(z)$ equals to

(1) $-\frac{\pi}{4}$ (2) $\frac{\pi}{4}$ (3) $-\frac{7\pi}{4}$ (4) none of these

62. The number of solutions to the equation $z^2+\bar{z}=0$ is equal to

(1) 1 (2) 2 (3) 3 (4) 4

63. i^{i} (when $i=\sqrt{-1}$) is

(1) a purely real number

(2) a purely complex number

(3) a complex number whose real part is always a negative real number

(4) a complex number whose real part is always a positive integer

64. The equation of the directrix of the parabola $y^2+4x+4y+2=0$ is

(1) $x=1$ (2) $x=-1$ (3) $x=\frac{3}{2}$ (4) $x=\frac{2}{3}$

65. Equation of the ellipse having vertices at (±5, 0) and foci at (±4, 0) is

(1) $\frac{x^2}{25}+\frac{y^2}{9}=1$ (2) $\frac{x^2}{25}+\frac{y^2}{16}=1$

(3) $\frac{x^2}{9}+\frac{y^2}{16}=1$ (4) $\frac{x^2}{4}+\frac{y^2}{5}=1$

66. The number of integer values of m, for which the x-co-ordinate of the point of intersection of the lines $x+y=3$ and $y=3mx+1$ is also an integer, is

(1) 0 (2) 1 (3) 2 (4) 4

67. The points $(a, 0)$, $(0, b)$ and $(1, -1)$ are collinear $(a \neq 0, b \neq 0)$ if

(1) $b-a=ab$ (2) $a+b=ab$

(3) $a-b=ab$ (4) $a+b=-ab$

68. If the points $(2a, a)$, $(a, 2a)$ and (a, a) form a triangle of area 32 sq. units, then the centroid of the triangle is

(1) (32, 32) (2) (−32, −32) (3) (3, 3) (4) $\left(\frac{32}{3}, \frac{32}{3}\right)$

69. If the curve $x^2+y^2-2x-2y+1=0$ intersects or touches the co-ordinate axes at A and B, then equation of the straight line joining A and B is

(1) $x+y=\sqrt{2}$ (2) $x+y=1$

(3) $x-y=1$ (4) $x-y=\sqrt{2}$

70. If the system of equations $(k+1)x+8y=4k$ and $kx+(k+3)y=3k-1$ has infinitely many solutions, then k is equal to

(1) 0 (2) 1 (3) 3 (4) −3

71. The solution of the differential equation

$$\frac{dy}{dx}+\frac{y}{1+x^2}=\frac{e^x}{e^{\tan^{-1}x}},\ y(0)=1$$

is

(1) $y=e^{x-\tan^{-1}x}$ (2) $y=e^x\cdot\tan^{-1}x+1$

(3) $y=\tan^{-1}x+1$ (4) $y=e^{x+\tan^{-1}x}$

72. The value of the integral $\int_0^1 x(1-x)^{49}\,dx$ is equal to

(1) $\frac{1}{2550}$ (2) $\frac{1}{2500}$ (3) $\frac{10}{490}$ (4) $\frac{1}{49}$

73. The value of

$$\underset{x\to 0}{\text{Lt}}\ \frac{\int_0^x \sin(t^2)\,dt}{x^3}$$

is equal to

(1) 0 (2) 1 (3) 3 (4) none of these

74. The value of the integral

$$\int\frac{e^x(1+\sin x)}{1+\cos x}\,dx$$

is equal to (k is any constant)

(1) $\log_e|\tan x|+k$ (2) $e^x\tan\left(\frac{x}{2}\right)+k$

(3) $e^x\tan x+k$ (4) $e^x\log_e|\sec x|+k$

75. The area bounded by the curves $y=\sqrt{x}$, $2y+3=x$ and the x-axis lying in the first quadrant, is (area in sq. units)

(1) 9 (2) 27 (3) $\frac{27}{2}$ (4) 18

Space For Rough Work

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